S2 Hypothesis Testing and Estimations from Samples Past Qu.s

1) June 2004 no.4

In a blood test, some blood is placed on a microscope slide and the number of corpuscles in each grid square of the slide is counted. The number of corpuscles per grid square in a sample of blood is a random variable with the distribution $Po(\mu)$.

(i) For healthy blood, it is known that $\mu = 2.0$. Find the probability that, in a randomly chosen sample of healthy blood, the number of corpuscles counted in one grid square is less than 3. [2]

(ii) A significance test of the null hypothesis $H_0 : \mu = 2.0$ as opposed to the alternative hypothesis $H_1 : \mu < 2.0$ is carried out, using a significance level as close as possible to 5%. The test is based on the total number of corpuscles counted in a group of 4 grid squares.

(a) Find the largest total number of corpuscles counted that would result in rejection of the null hypothesis. You should show the value of any relevant probability. [2]

(b) Given that, in fact, $\mu = 1.75$, find the probability that the test results in a Type II error. [3]

2) Jan 2005 no.4

A local government spokesman claims that at least three-quarters of the residents of a town are in favour of plans to build a new by-pass for the town. An opinion poll showed that 10 out of a random sample of 16 residents of the town were in favour of the plans. Test, at the 10% significance level, whether the results of the opinion poll are consistent with the spokesman's claim, stating your hypotheses clearly. [7]

3) Jan 2005 no.6

(i) Explain what is meant by a Type I error. [1]

(ii) The continuous random variable X has the distribution $N(\mu, \sigma^2)$. A test of the hypothesis $H_0 : \mu = 25$ is carried out at the 5% significance level, once a day for 300 days. Given that on each day the value of μ is 25, use a normal approximation to find the probability that a Type I error is made on at least 20 days. [6]

(iii) Explain whether, in answering part (ii), it is necessary to assume that the outcomes of the tests are independent. [1]

4) Jan 2005 no. 8

(i) A random variable X has the distribution $N(\mu, \sigma^2)$. The mean of a sample of 5 observations of X is denoted by $\bar{X}$. State the distribution of $\bar{X}$, giving the values of any parameters. [2]

(ii) A group of scientists is attempting to identify subatomic particles called *ocrons*. Ocrons have a mean path length of less than 42 cm. The path lengths of a random sample of five particles thought to be ocrons are measured, and the mean path length of the sample is found to be 36.6 cm. Path lengths are known to be normally distributed random variables with standard deviation 8 cm. Carry out a test, at the 10% significance level, of whether the population mean path length is less than 42 cm, stating your hypotheses clearly. [7]

(iii) A second group of scientists carries out a test that is identical, except that they use a 5% significance level. If the mean observed path length of the particles is consistent with a population mean of less than 42 cm, the scientists will claim that the particles are ocrons. State what the use of this smaller significance level suggests about the intentions of the scientists in deciding whether or not to claim that the observed particles are ocrons. [2]

5) June 2005 no.4

The height of sweet pea plants grown in a nursery is a random variable. A random sample of 50 plants is measured and is found to have a mean height 1.72 m and variance 0.0967 m^2.

(i) Calculate an unbiased estimate for the population variance of the heights of sweet pea plants. [2]

(ii) Hence test, at the 10% significance level, whether the mean height of sweet pea plants grown by the nursery is 1.8 m, stating your hypotheses clearly. [7]

6) June 2005 no.6

A factory makes chocolates of different types. The proportion of milk chocolates made on any day is denoted by p. It is desired to test the null hypothesis $H_0 : p = 0.8$ against the alternative hypothesis $H_1 : p < 0.8$. The test consists of choosing a random sample of 25 chocolates. H_0 is rejected if the number of milk chocolates is k or fewer. The test is carried out at a significance level as close to 5% as possible.

(i) Use tables to find the value of k, giving the values of any relevant probabilities. [3]

(ii) The test is carried out 20 times, and each time the value of p is 0.8. Each of the tests is independent of all the others. State the expected number of times that the test will result in rejection of the null hypothesis. [2]

(iii) The test is carried out once. If in fact the value of p is 0.6, find the probability of rejecting H_0. [2]

(iv) The test is carried out twice. Each time the value of p is equally likely to be 0.8 or 0.6. Find the probability that exactly one of the two tests results in rejection of the null hypothesis. [4]

7) June 2005 no.7

In excavating an archaeological site, Roman coins are found scattered throughout the site.

(i) State two assumptions needed to model the number of coins found per square metre of the site by a Poisson distribution. [2]

Assume now that the number of coins found per square metre of the site can be modelled by a Poisson distribution with mean λ.

(ii) Given that $\lambda = 0.75$, calculate the probability that exactly 3 coins are found in a region of the site of area $7.20\,\text{m}^2$. [3]

A test is carried out, at the 5% significance level, of the null hypothesis $\lambda = 0.75$, against the alternative hypothesis $\lambda > 0.75$, in Region LVI which has area $4\,\text{m}^2$.

(iii) Determine the smallest number of coins that, if found in Region LVI, would lead to rejection of the null hypothesis, stating also the values of any relevant probabilities. [4]

(iv) Given that, in fact, $\lambda = 1.2$ in Region LVI, find the probability that the test results in a Type II error. [3]

8) Jan 2006 no.3

The manufacturers of a brand of chocolates claim that, on average, 30% of their chocolates have hard centres. In a random sample of 8 chocolates from this manufacturer, 5 had hard centres. Test, at the 5% significance level, whether there is evidence that the population proportion of chocolates with hard centres is not 30%, stating your hypotheses clearly. Show the values of any relevant probabilities. [7]

9) Jan 2006 no.7

The random variable X has the distribution $N(\mu, 8^2)$. The mean of a random sample of 12 observations of X is denoted by $\bar{X}$. A test is carried out at the 1% significance level of the null hypothesis H_0: $\mu = 80$ against the alternative hypothesis H_1: $\mu < 80$. The test is summarised as follows: 'Reject H_0 if $\bar{X} < c$; otherwise do not reject H_0'.

(i) Calculate the value of c. [4]

(ii) Assuming that $\mu = 80$, state whether the conclusion of the test is correct, results in a Type I error, or results in a Type II error if:

(a) $\bar{X} = 74.0$, [1]

(b) $\bar{X} = 75.0$. [1]

(iii) Independent repetitions of the above test, using the value of c found in part **(i)**, suggest that in fact the probability of rejecting the null hypothesis is 0.06. Use this information to calculate the value of μ. [4]

10) June 2006 no.2

(i) The random variable R has the distribution $B(6, p)$. A random observation of R is found to be 6. Carry out a 5% significance test of the null hypothesis H_0: $p = 0.45$ against the alternative hypothesis H_1: $p \neq 0.45$, showing all necessary details of your calculation. [4]

(ii) The random variable S has the distribution $B(n, p)$. H_0 and H_1 are as in part **(i)**. A random observation of S is found to be 1. Use tables to find the largest value of n for which H_0 is not rejected. Show the values of any relevant probabilities. [3]

11) June 2006 no.7

Three independent researchers, A, B and C, carry out significance tests on the power consumption of a manufacturer's domestic heaters. The power consumption, X watts, is a normally distributed random variable with mean μ and standard deviation 60. Each researcher tests the null hypothesis H_0: $\mu = 4000$ against the alternative hypothesis H_1: $\mu > 4000$.

Researcher A uses a sample of size 50 and a significance level of 5%.

(i) Find the critical region for this test, giving your answer correct to 4 significant figures. [6]

In fact the value of μ is 4020.

(ii) Calculate the probability that Researcher A makes a Type II error. [6]

(iii) Researcher B uses a sample bigger than 50 and a significance level of 5%. Explain whether the probability that Researcher B makes a Type II error is less than, equal to, or greater than your answer to part **(ii)**. [2]

(iv) Researcher C uses a sample of size 50 and a significance level bigger than 5%. Explain whether the probability that Researcher C makes a Type II error is less than, equal to, or greater than your answer to part **(ii)**. [2]

(v) State with a reason whether it is necessary to use the Central Limit Theorem at any point in this question. [2]

12) Jan 2007 no.7

A television company believes that the proportion of households that can receive Channel C is 0.35.

(i) In a random sample of 14 households it is found that 2 can receive Channel C. Test, at the 2.5% significance level, whether there is evidence that the proportion of households that can receive Channel C is less than 0.35. [7]

(ii) On another occasion the test is carried out again, with the same hypotheses and significance level as in part **(i)**, but using a new sample, of size n. It is found that no members of the sample can receive Channel C. Find the largest value of n for which the null hypothesis is not rejected. Show all relevant working. [4]

13) Jan 2007 no.8

The quantity, X milligrams per litre, of silicon dioxide in a certain brand of mineral water is a random variable with distribution $N(\mu, 5.6^2)$.

(i) A random sample of 80 observations of X has sample mean 100.7. Test, at the 1% significance level, the null hypothesis $H_0 : \mu = 102$ against the alternative hypothesis $H_1 : \mu \neq 102$. [5]

(ii) The test is redesigned so as to meet the following conditions.

- The hypotheses are $H_0 : \mu = 102$ and $H_1 : \mu < 102$.
- The significance level is 1%.
- The probability of making a Type II error when $\mu = 100$ is to be (approximately) 0.05.

The sample size is n, and the critical region is $\overline{X} < c$, where $\overline{X}$ denotes the sample mean.

(a) Show that n and c satisfy (approximately) the equation $102 - c = \dfrac{13.0256}{\sqrt{n}}$. [3]

(b) Find another equation satisfied by n and c. [2]

(c) Hence find the values of n and c. [4]

14) June 2007 no.5

The number of system failures per month in a large network is a random variable with the distribution $Po(\lambda)$. A significance test of the null hypothesis $H_0 : \lambda = 2.5$ is carried out by counting R, the number of system failures in a period of 6 months. The result of the test is that H_0 is rejected if $R > 23$ but is not rejected if $R \leqslant 23$.

(i) State the alternative hypothesis. [1]

(ii) Find the significance level of the test. [3]

(iii) Given that $P(R > 23) < 0.1$, use tables to find the largest possible actual value of λ. You should show the values of any relevant probabilities. [3]

15) June 2007 no.6

In a rearrangement code, the letters of a message are rearranged so that the frequency with which any particular letter appears is the same as in the original message. In ordinary German the letter *e* appears 19% of the time. A certain encoded message of 20 letters contains one letter *e*.

(i) Using an exact binomial distribution, test at the 10% significance level whether there is evidence that the proportion of the letter *e* in the language from which this message is a sample is less than in German, i.e., less than 19%. [8]

(ii) Give a reason why a binomial distribution might not be an appropriate model in this context. [1]

16) June 2007 no.8

A random variable Y is normally distributed with mean μ and variance 12.25. Two statisticians carry out significance tests of the hypotheses $H_0 : \mu = 63.0$, $H_1 : \mu > 63.0$.

(i) Statistician A uses the mean $\overline{Y}$ of a sample of size 23, and the critical region for his test is $\overline{Y} > 64.20$. Find the significance level for A's test. [4]

(ii) Statistician B uses the mean of a sample of size 50 and a significance level of 5%.

(a) Find the critical region for B's test. [3]

(b) Given that $\mu = 65.0$, find the probability that B's test results in a Type II error. [4]

(iii) Given that, when $\mu = 65.0$, the probability that A's test results in a Type II error is 0.1365, state with a reason which test is better. [2]

17) Jan 2008 no. 3

The random variable G has the distribution $Po(\lambda)$. A test is carried out of the null hypothesis $H_0 : \lambda = 4.5$ against the alternative hypothesis $H_1 : \lambda \neq 4.5$, based on a single observation of G. The critical region for the test is $G \leqslant 1$ and $G \geqslant 9$.

(i) Find the significance level of the test. [5]

(ii) Given that $\lambda = 5.5$, calculate the probability that the test results in a Type II error. [3]

18) Jan 2008 no.5

Over a long period the number of visitors per week to a stately home was known to have the distribution $N(500, 100^2)$. After higher car parking charges were introduced, a sample of four randomly chosen weeks gave a mean number of visitors per week of 435. You should assume that the number of visitors per week is still normally distributed with variance 100^2.

(i) Test, at the 10% significance level, whether there is evidence that the mean number of visitors per week has fallen. [7]

(ii) Explain why it is necessary to assume that the distribution of the number of visitors per week (after the introduction of higher charges) is normal in order to carry out the test. [2]

19) Jan 2008 no.8

Consultations are taking place as to whether a site currently in use as a car park should be developed as a shopping mall. An agency acting on behalf of a firm of developers claims that at least 65% of the local population are in favour of the development. In a survey of a random sample of 12 members of the local population, 6 are in favour of the development.

(i) Carry out a test, at the 10% significance level, to determine whether the result of the survey is consistent with the claim of the agency. [7]

(ii) A local residents' group claims that no more than 35% of the local population are in favour of the development. Without further calculations, state with a reason what can be said about the claim of the local residents' group. [2]

(iii) A test is carried out, at the 15% significance level, of the agency's claim. The test is based on a random sample of size $2n$, and exactly n of the sample are in favour of the development. Find the smallest possible value of n for which the outcome of the test is to reject the agency's claim. [4]

20) June 2008 no.4

The random variable U has the distribution $N(\mu, \sigma^2)$, where the value of σ is known. A test is carried out of the null hypothesis $H_0 : \mu = 50$ against the alternative hypothesis $H_1 : \mu > 50$. The test is carried out at the 1% significance level and is based on a random sample of size 10.

(i) The test is carried out once. The value of the sample mean is 53.0. The outcome of the test is that H_0 is not rejected. Show that $\sigma > 4.08$, correct to 3 significant figures. [4]

(ii) The test is carried out repeatedly. In each test the actual value of μ is 50. Find the probability that the first test to result in a Type I error is the fifth to be carried out. Give your answer correct to 2 significant figures. [3]

21) June 2008 no.7

Wendy analyses the number of 'dropped catches' in international cricket matches. She finds that the mean number of dropped catches per day is 2. In a recent 5-day match she found that there was a total of c dropped catches. She tests, at the 5% significance level, whether the mean number of dropped catches per day has increased.

(i) State conditions needed for the number of dropped catches per day to be well modelled by a Poisson distribution. [2]

Assume now that these conditions hold.

(ii) Find the probability that the test results in a Type I error.

(iii) Given that $c = 14$, carry out the test.

[10]

22) June 2008 no.8

A company sponsors a series of concerts. Surveys show that on average 40% of audience members know the name of the sponsor. As this figure is thought to be disappointingly low, the publicity material is redesigned.

(i) After the publicity material has been redesigned, a random sample of 12 audience members is obtained, and it is found that 9 members of this sample know the name of the sponsor. Test, at the 5% significance level, whether there is evidence that the proportion of audience members who know the name of the sponsor has increased. [7]

(ii) A more detailed 5% hypothesis test is carried out, based on a random sample of size 400. This test produces significant evidence that the proportion of audience members knowing the name of the sponsor has increased. Using an appropriate approximation, calculate the smallest possible number of audience members in the sample of 400 who know the name of the sponsor. [7]

23) Jan 2009 no.4

A television company believes that the proportion of adults who watched a certain programme is 0.14. Out of a random sample of 22 adults, it is found that 2 watched the programme.

(i) Carry out a significance test, at the 10% level, to determine, on the basis of this sample, whether the television company is overestimating the proportion of adults who watched the programme. **[8]**

(ii) The sample was selected randomly. State what properties of this method of sampling are needed to justify the use of the distribution used in your test. **[2]**

24) Jan 2009 no.6

The weight of a plastic box manufactured by a company is W grams, where $W \sim \mathrm{N}(\mu, 20.25)$. A significance test of the null hypothesis $\mathrm{H}_0 : \mu = 50.0$, against the alternative hypothesis $\mathrm{H}_1 : \mu \neq 50.0$, is carried out at the 5% significance level, based on a sample of size n.

(i) Given that $n = 81$,

(a) find the critical region for the test, in terms of the sample mean $\overline{W}$, **[5]**

(b) find the probability that the test results in a Type II error when $\mu = 50.2$. **[5]**

(ii) State how the probability of this Type II error would change if n were greater than 81. **[1]**

25) June 2009 no.8

In a large company the time taken for an employee to carry out a certain task is a normally distributed random variable with mean 78.0 s and unknown variance. A new training scheme is introduced and after its introduction the times taken by a random sample of 120 employees are recorded. The mean time for the sample is 76.4 s and an unbiased estimate of the population variance is 68.9 s^2.

(i) Test, at the 1% significance level, whether the mean time taken for the task has changed. **[7]**

(ii) It is required to redesign the test so that the probability of making a Type I error is less than 0.01 when the sample mean is 77.0 s. Calculate an estimate of the smallest sample size needed, and explain why your answer is only an estimate. **[4]**

26) Jan 2010 no.8

The random variable R has the distribution $\mathrm{B}(10, p)$. The null hypothesis H_0: $p = 0.7$ is to be tested against the alternative hypothesis H_1: $p < 0.7$, at a significance level of 5%.

(i) Find the critical region for the test and the probability of making a Type I error. **[3]**

(ii) Given that $p = 0.4$, find the probability that the test results in a Type II error. **[3]**

(iii) Given that p is equally likely to take the values 0.4 and 0.7, find the probability that the test results in a Type II error. **[2]**

27) June 2010 no.5

The time T seconds needed for a computer to be ready to use, from the moment it is switched on, is a normally distributed random variable with standard deviation 5 seconds. The specification of the computer says that the population mean time should be not more than 30 seconds.

(i) A test is carried out, at the 5% significance level, of whether the specification is being met, using the mean $\bar{t}$ of a random sample of 10 times.

(a) Find the critical region for the test, in terms of $\bar{t}$. **[4]**

(b) Given that the population mean time is in fact 35 seconds, find the probability that the test results in a Type II error. **[3]**

(ii) Because of system degradation and memory load, the population mean time μ seconds increases with the number of months of use, m. A formula for μ in terms of m is $\mu = 20 + 0.6m$. Use this formula to find the value of m for which the probability that the test results in rejection of the null hypothesis is 0.5. **[4]**

28) June 2010 no.3

80 randomly chosen people are asked to estimate a time interval of 60 seconds without using a watch or clock. The mean of the 80 estimates is 58.9 seconds. Previous evidence shows that the population standard deviation of such estimates is 5.0 seconds. Test, at the 5% significance level, whether there is evidence that people tend to underestimate the time interval. **[7]**

29) June 2009 no.3

An electronics company is developing a new sound system. The company claims that 60% of potential buyers think that the system would be good value for money. In a random sample of 12 potential buyers, 4 thought that it would be good value for money. Test, at the 5% significance level, whether the proportion claimed by the company is too high. **[7]**

30) Jan 2010 no.4

The proportion of commuters in a town who travel to work by train is 0.4. Following the opening of a new station car park, a random sample of 16 commuters is obtained, and 11 of these travel to work by train. Test at the 1% significance level whether there is evidence of an increase in the proportion of commuters in this town who travel to work by train. **[7]**

31) Jan 2010 no.5

The number of customers arriving at a store between 8.50 am and 9 am on Saturday mornings is a random variable which can be modelled by the distribution Po(11.0). Following a series of price cuts, on one particular Saturday morning 19 customers arrive between 8.50 am and 9 am. The store's management claims, first, that the mean number of customers has increased, and second, that this is due to the price cuts.

(i) Test the first part of the claim, at the 5% significance level. **[7]**

(ii) Comment on the second part of the claim. **[1]**